Foundations of Discrete Mathematics
COT 2104
Summer 2008

Homework 2

NOTE: This homework will be collected in three stages. The corresponding deadline appears before each set of questions. The final grade of this homework will covered the results of the three stages.

Deadline: 06/16/08 (at the beginning of class)

1. Determine whether or not each of the following relations is a function with domain {1, 2, 3, 4}. For any relation that is not a function, explain why it isn't.

   a) f = {(1, 2), (2, 3), (4, 2)}
   b) f = {(1, 1), (1, 2), (1, 3), (1, 4)}
   c) f = {(1, 4), (2, 3), (3, 2), (4, 1)}

2. Suppose A is the set of students registered at the University of Calgary, and C is the set of courses currently being offered at the University of Calgary. Under what conditions is each of the following a function?
   a) {(a, b) | a's first class each week is in c}
   b) {(a, b) | a has a class in c Saturday evening}

3. Let A = {1, 2, 3, 4}. Find the inverse of the following function f: A → A.
   a) f = {(1, 2), (2, 4), (3, 3), (4, 1)}

4. Given the following function find its inverse. Specify the domain and range of the inverse.
   a) f: R → R given by $f(x) = x^3 - 2$

5. Show that the function f : A → R is one-to-one. Find the range.
   A = { x ∈ R | x ≠ -1}, f(x) = 5 – 1 / (1 + x)

6. Find a one-to-one correspondence between each of the following pairs of sets:

   a) {x, y, {a, b, c}} and {14, -3, t}
   b) 2Z and 17Z

7. Let f = {(1, 2), (2, 3), (3, 4) , (4, 1) and g = {(1, 3), (2, 1), (3, 4), (4, 2), (5, 1)}. Find f^{-1} and g ° f. Is g one-to-one? Explain.

Deadline: 06/23/08(at the beginning of class)

8. Determine whether or not each of the following integers is a prime
   a) 9833
   b) 150

9. Find q and r as defined by the division algorithm

   a) a = -5286; b = 19
   b) a = 5286; b = -19

10. Find the binary, octal and hexadecimal representations for each of the following integers (given in base 10).

    a) 57,483
    b) 185,178

11. In each of the following cases, find the greatest common divisor (gcd) of a and b applying the Euclidean algorithm.

    a) a = 1575, b = 231
    b) a = -3719, b = 8416
    c) a = 28,844, b = -15,712

12. Find the least common multiple (lcm) of the pairs of integers given in exercises 10.

13. Find the prime numbers less than or equal to the following natural numbers.

    a) less than 200
    b) less than 500

14. Find a (mod n) in each of the following cases.

    a) a = 43,197, n = 39
    b) a = -125,617, n = 315

15. Find a solution in each of the following congruencies

    a) $x \equiv 7 \pmod 9$
    b) $x \equiv 4 \pmod{12}$
    c) $x \equiv 16 \pmod{21}$

Deadline: 06/30/08 (at the beginning of the test)

16. Write each of the following sums without using $\sum$ and evaluate.

    a) $\sum_{j=0}^{2} 3^{j+2}$
    b) $\sum_{k=0}^{n} (-1)^k$

17. Use mathematical induction to prove the truth of each of the following assertions for all $n \geq 1$.

    a) $n^3 + 2n$ is divisible by 3
    b) $5^n - 1$ is divisible by 4

18. Use mathematical induction to establish the following formula.

    a) $\sum_{i=1}^{n} i^2 / ((2i - 1)(2i + 1)) = n(n+ 1)/(2(2n + 1))$

19. Give recursive definitions of each of the following sequences:

    a) 5, 3, 1, -1, -3, …
    b) 1, 2, 0, 3, -1, 4, -2, …

20. Solve the recurrence relation $a_n = -6a_{n-1} + 7a_{n-2}$, $n \geq 2$, given $a_0 = 32$, $a_1 = -17$.

21. Express the generating function of each of the following sequences as a polynomial or as the quotient of polynomial:

    a) 0, 1, 4, 1, 0, 0, …
    b) 3, 3, 3, …

22. Consider the arithmetic sequence begins 5, 9, 13.

    a) Find the 32^{nd}. and 100^{th}. terms of this sequence.